Math 124 - Section 012
Hints for section 4.6

13(b) By the chain rule, $\frac{dx}{dt} = \frac{dx}{d\theta}\frac{d\theta}{dt}$. Compute $\frac{dx}{d\theta}$ using the formula $x = \cos\theta$. From the graph, $\frac{d\theta}{dt}|_{t=2} \approx 1$. Compute $\frac{dy}{dt}$ in the same way (using $y = \sin\theta$).

18(c) Combine the equations $\frac{r}{h} = \frac{6}{10}$ and $V = \frac{1}{3}\pi r^2 h$ to get an equation for V as a function of h. Differentiate this equation with respect to t (using the chain rule: $\frac{dV}{dt} = \frac{dV}{dh}\frac{dh}{dt}$), and solve for $\frac{dh}{dt}$ (as a function of h). You will need to use $\frac{dV}{dt} = -1.5$.

20(b) Let V=volume, A=area, h=thickness. So $V = A \cdot h$. Differentiate this equation with respect to t (use the product rule on the right-hand side). Using $\frac{dV}{dt} = 0$ and part (a), you can now solve for $\frac{dh}{dt}$.

22 Think of x and y as functions of t, and differentiate the equation $x^2 + y^2 = 62.5$ with respect to t. Then solve for $\frac{dx}{dt}$. You will need to use the fact that in one minute, the wheel travels through $\frac{2\pi}{20}$ radians.

24(c) Using $\frac{x}{.5} = \tan\theta$, we have $\theta = \arctan\left(\frac{x}{.5}\right)$. Compute $\frac{d\theta}{dt}$ using the chain rule.

30 Using $r = h \cdot \tan(\frac{\pi}{6} = \frac{h}{\sqrt{3}}$, we get $V = \frac{1}{9}\pi h^2$. Compute $\frac{dV}{dh}$. By the chain rule, $\frac{dV}{dt} = \frac{dV}{dh}\frac{dh}{dt}$. Solve for $\frac{dh}{dt}$.

33(b) From the picture, $\tan\theta = \frac{h(t)-100}{150}$. Solve for θ and compute.

33(c) To find the point at which $\frac{d\theta}{dt}$ is maximal, you could compute $\frac{d^2\theta}{dt^2}$, find the critical point(s), etc., but the formula you got in part (b) should be simple enough that the answer can be computed without calculus.

35(e) $y'(t) = -k \cdot x(t)$ where k is positive.

35(f) Differentiate $x^2(t) - y^2(t) = 900$ with respect to t and use part (e). Now solve for $x'(t)$.

35(g) From part (b), $x(3) = 34$ and $y(3) = 16$. We are now given that $x'(3) = 32$. Substitute these values into the equation you found in part (f) to find k. Now use part (e) to find $y'(3)$.